Huyber's Model of Glacial Cycles

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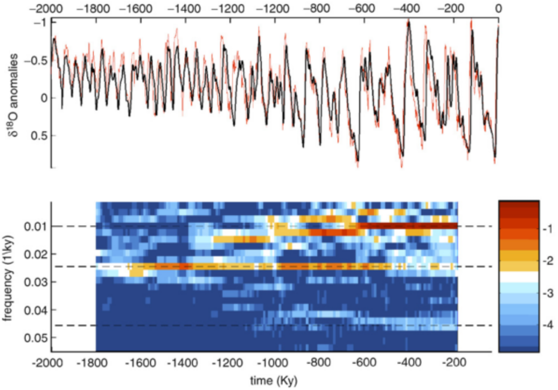
Huybers' Model of Glacial Cycles

Huybers, P. Glacial variability over the last two million years: an extended depth-derived agemodel, continuous obliquity pacing, and the Pleistocene progression. *Quaternary Science Reviews*. 2007.

The Obliquity Hypothesis

- H_0: Deglaciations are independent of obliquity.
- H_1: Deglaciations are triggered at a particular phase of Earth's obliquity.



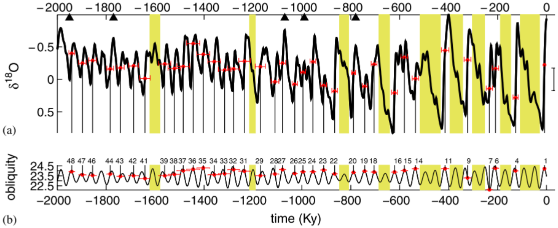
δ^{18}O record of last 2 Ma

Rayleigh's R

- ϕ_n is the phase of obliquity sampled at the n^{th} deglacial event.
- Rayleigh's R

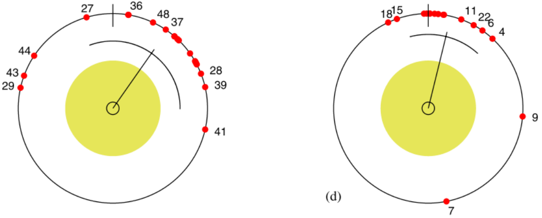
$$R = \frac{1}{N}\left|\sum_{n=1}^{N} \cos\phi_n + i\sin\phi_n\right|$$

Deglacial events



Deglacial events and corresponding obliquity cycle

Rayleigh circles

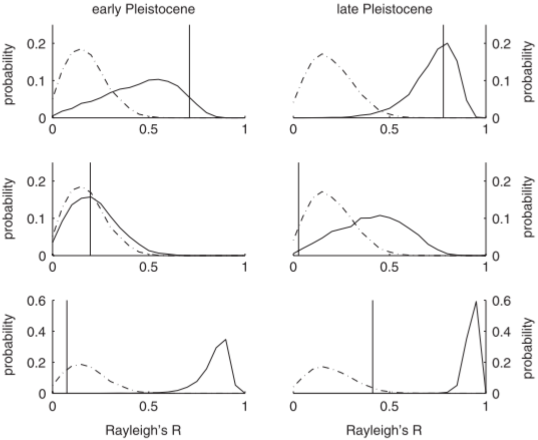


Rayleigh circles with obliquity phases plotted for early Pleistocene (left) and late Pleistocene (right)

Rayleigh R vs. Fourier analysis

- Nonlinearities associated with the variable duration and asymmetry do not affect the statistic.
- Errors in agemodel cause only linear changes in the phase, but distort the Fourier spectrum
- Requires fewer data points for statistical significance

Rayleigh Statistic Results



Top: obliquity, middle: precession, bottom: eccentricity. The vertical line indicates the R value. Dashed lines indicate the probability distribution for H_0; solid lines give the probability distribution for H_1.

Rayleigh Statistic Results

	Early Pleistocene (2–1 Ma)				Late Pleistocene (1–0 Ma)			
	R	cv 1%	Power	Phase	R	cv 1%	Power	Phase
Obliquity	0.7	0.5	0.6	$\pm 56^\circ$	0.8	0.5	1.0	$\pm 28^\circ$
Precession	0.2	0.5	0.0	$\pm 88^\circ$	0.0	0.5	0.3	$\pm 56^\circ$
Eccentricity	0.1	0.5	1.0	$\pm 24^\circ$	0.4	0.5	1.0	$\pm 12^\circ$

Null hypothesis is rejected only for obliquity, implying that deglaciations are triggered at a particular phase of obliquity.

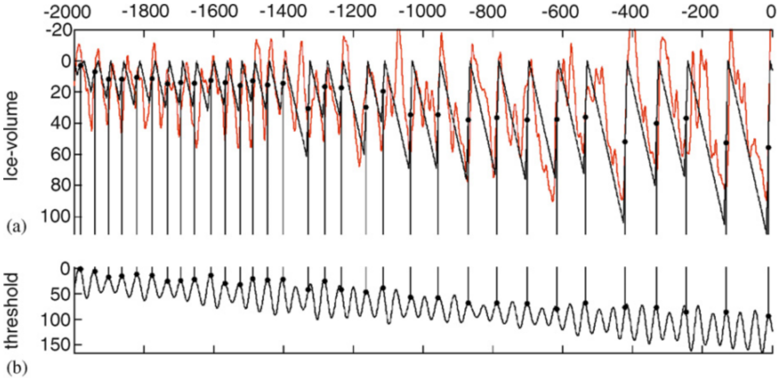
- The late Pleistocene deglacial events have $R = 0.8$ vs. $R = 0.7$ in the early Pleistocene.
- The late Pleisotcene 100 Ka world has *greater* obliquity phase stability than the early Pleistocene 40 Ka world.

Huybers' Model

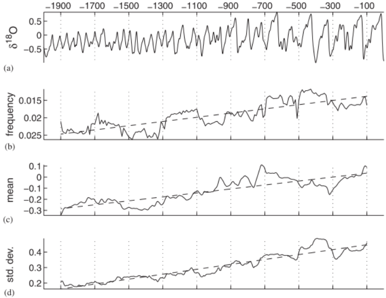
$$\begin{aligned} V_t &= V_{t-1} + \eta_t \quad \text{and if } V_t \geqslant T_t \text{ terminate} \\ T_t &= at + b - c\theta'_t \end{aligned} \tag{1}$$

Upon termination, linearly reset V to 0 over 10 Ka

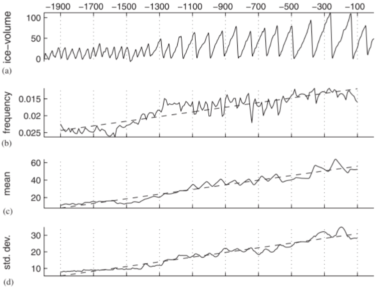
V	:	ice volume
T	:	deglaciation threshold
θ'	:	scaled obliquity
η	:	ice volume growth rate



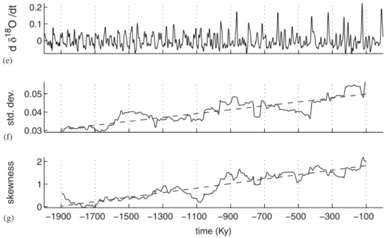
A deterministic run of the model



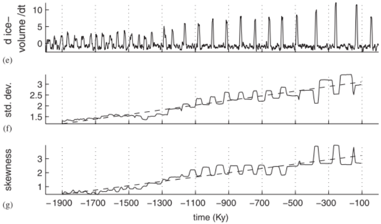
δO^{18} data



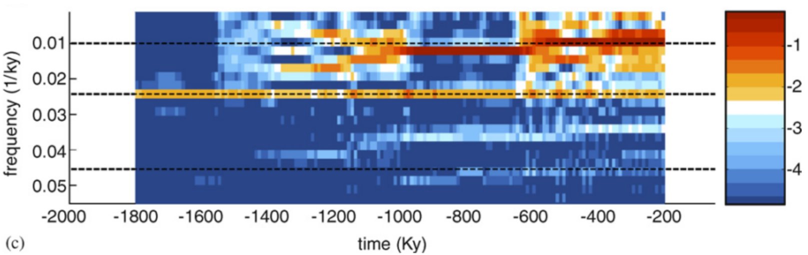
Modeled data



δO^{18} data



Modeled data

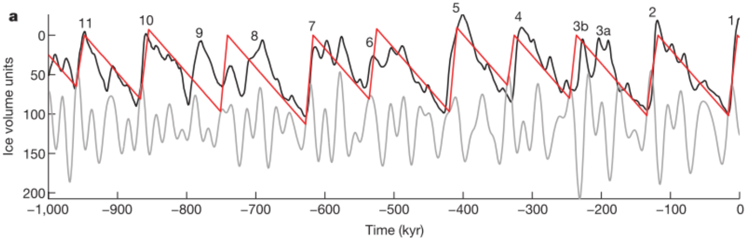


Fourier transform of stochastic model

Deglaciation Model with Combined Forcing

Huybers, P. Combined obliquity and precession pacing of late Pleistocene deglaciations. *Nature*. 2011.

$$
\begin{aligned}
V_t &= V_{t-1} + \eta_t \qquad \text{and if } V_t \geqslant T_t \text{ terminate} \\
T_t &= 110 - 25\mathcal{F}_t \\
\mathcal{F}_t &= \alpha^{1/2} e_t \sin(\omega_t - \phi) + (1-\alpha)^{1/2} \epsilon_t
\end{aligned}
$$



Deterministic model with combined forcing

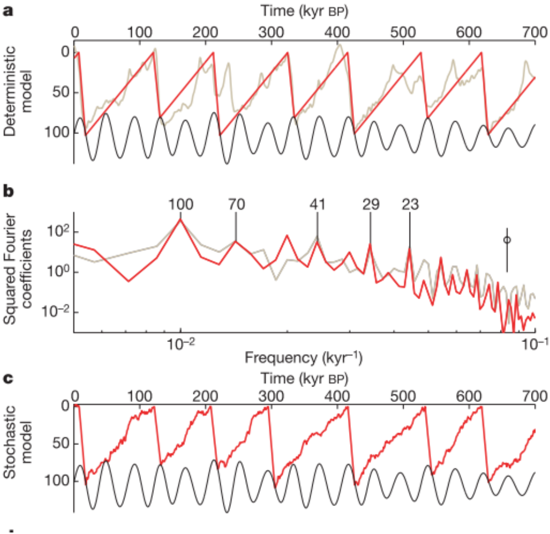
Conclusion from 2011 paper

"Precession will tend to influence the precise timing of a deglaciation cycle, but obliquity will more fundamentally govern the interval between glaciations."

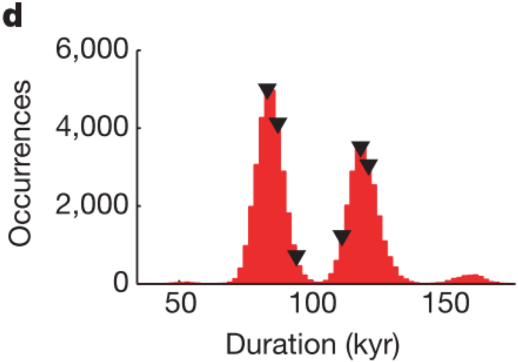
Huybers and Wunsch original paper

Huybers, P. and Wunsch, C. Obliquity pacing of the late Pleistocene glacial terminations. *Nature*. 2005.

$$
\begin{aligned}
V_t &= V_{t-1} + \eta_t \quad \text{and if } V_t \geqslant T_t \text{ terminate} \\
T_t &= 100 - \theta'_t
\end{aligned}
$$



Deterministic and stochastic models with obliquity forcing



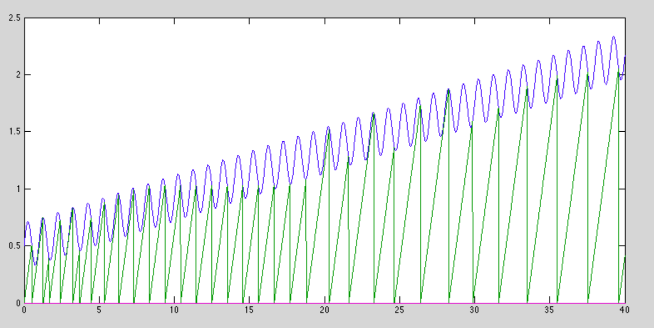
Histogram of time between terminations for many runs of stochastic model

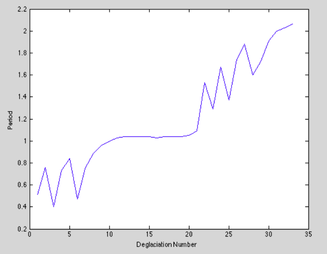
Mathematical Questions

What happens in an idealized case when the forcing is just a sine curve?

$$\begin{aligned} V_t &= V_{t-1} + \eta_t \qquad \text{and if } V_t \geqslant T_t \text{ terminate} \\ T_t &= at + b + c\sin(2\pi t) \end{aligned}$$

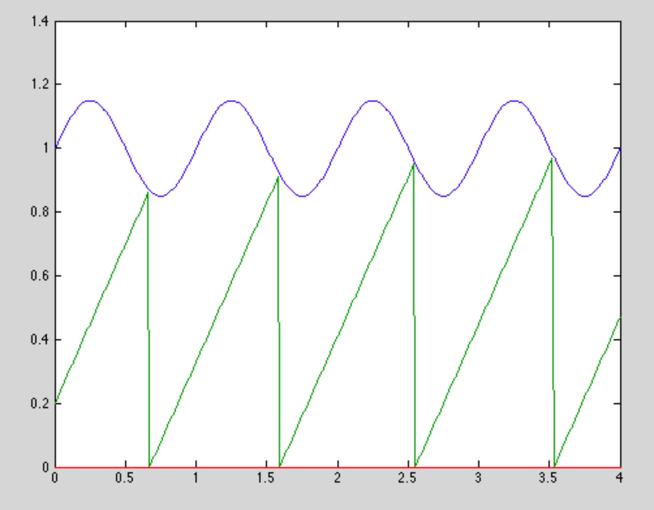
Matlab simulation with sinusoidal obliquity





Progression of the period

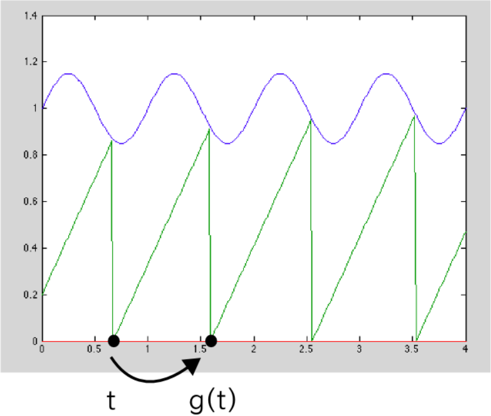
Simpler case: no increase over time



Suppose the threshold $T(t)$ is periodic with period 1:

$$T(t+1) = T(t)$$

Let $g : \mathbb{R} \to \mathbb{R}$ be the map taking a termination time t to the next termination time:



Suppose we start at a termination time x.
The next termination is at

$$g(x) = y \quad \text{where } y - x = T(y).$$

Then

$$T(y+1) = T(y) = y - x = (y+1) - (x+1)$$

So $y + 1$ would be next termination starting from time $x + 1$:

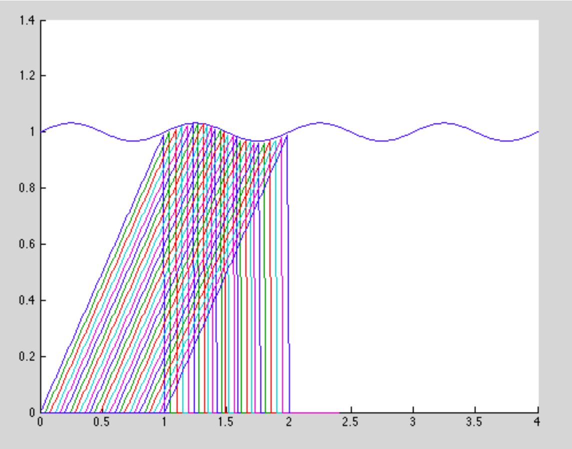
$$g(x+1) = y + 1 = g(x) + 1$$

This means that

$$g(x+1) = g(x) \mod 1$$

We can treat this simple model as a circle map $f : [0, 1) \to [0, 1)$:

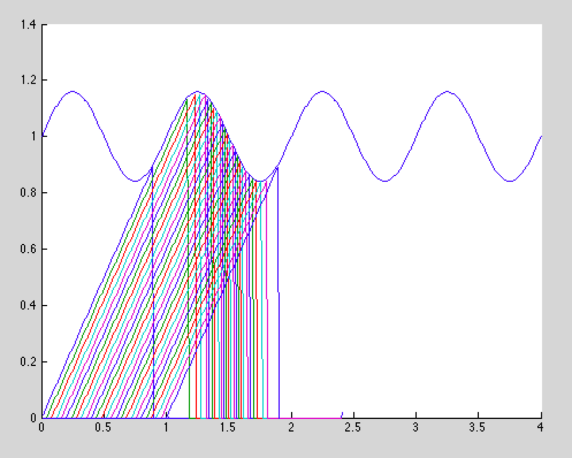
$$f(x) = y \mod 1 \qquad \text{where } y - x = A\sin(y) + B = T(y)$$



Smooth

We can treat this simple model as a circle map $f : [0,1) \to [0,1)$:

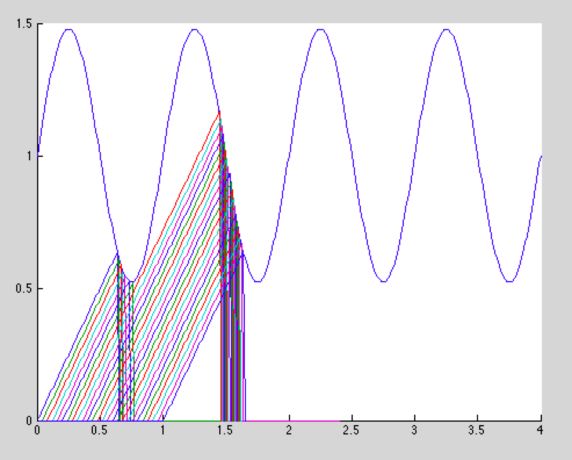
$$f(x) = y \mod 1 \qquad \text{where } y - x = A\sin(y) + B = T(y)$$



Continuous

We can treat this simple model as a circle map $f : [0, 1) \to [0, 1)$:

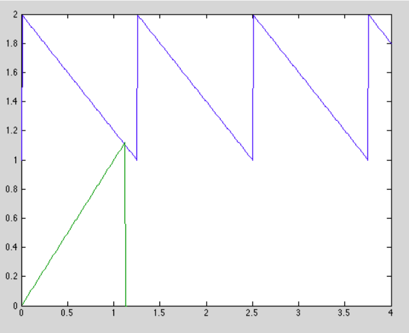
$$f(x) = y \mod 1 \qquad \text{where } y - x = A\sin(y) + B = T(y)$$



Discontinuous

Even simpler circle maps

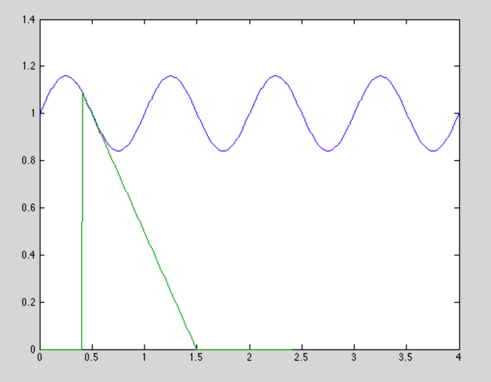
$$f(x) = mx + b \mod 1$$



- Canonical translation if $m = 1$
- Not surjective if $m < 1$
- Not injective if $m > 1$

Standard family of circle maps

$$f(x) = x + b + \frac{\omega}{2\pi}\sin(2\pi x) \mod 1$$



Questions

- What is $\lim_{n\to\infty} f^n(x)$ or $\lim_{n\to\infty} f^n([0,1])$?
- Is it possible to classify these maps by rotation numbers?
- Relation to Arnold tongues?
- Can we describe the transitions by increasing the threshold in Huybers' model?